21 Lessons from the International PISA Project

Ross Turner

A simple characterisation of "the good old days": All students in the early years of secondary school studied Mathematics. It was usually taught in a rigorous and challenging way that functioned in part as a sorting mechanism. Most students found it difficult, and eventually gave up studying Mathematics. Many of these ended up hating Mathematics and feeling they were no good at it. And for many, this turned in later life into an attitude of fear and anxiety about Math, and a wish to avoid it where possible. A small proportion of students loved Mathematics, and did well at it. They were part of the relatively small cohort of students who continued through to the end of secondary schooling. Some of these went on with their studies at university level, during which they developed higher level Mathematics skills useful in scientific, technical or quantitative areas.

What has changed? Well, many things of course, of which I mention two. We have a far greater proportion of students continuing their schooling at least to the end of secondary school. And there is far greater demand for quantitative competence in a wide variety of occupations. The level of technical skill including mathematical competence required in the modern workforce continues to expand. The more formal and traditional mathematical skills continue to be important, but it should also be acknowledged that our whole world demands a level of quantitative competence and confidence, to meet even simple day to day challenges in the workplace and in managing our personal circumstances, that is a quantum leap from the simpler demands of the good old days. Increasingly in our world, people who fear or are otherwise incapable of handling mathematical challenges would seem to be at a severe disadvantage. Are there any implications of these developments for schools? Increased school retention rates give schools a key role in equipping their students to better handle these challenges.

But my question is this: to what extent have our teaching and learning practices, our curriculum decisions and our approaches to assessment, changed to accommodate these new demands? The Organisation for Economic Cooperation and Development (OECD) Programme for International Student Assessment (PISA) is a comparative survey designed to assess the extent to which students at the age of 15 in participating countries, who are nearing the end of compulsory schooling, are prepared to meet the challenges of the modern world. PISA aims to measure the extent to which students are able to use the knowledge and skills they have acquired throughout their schooling to meet challenges that involve activation of reading, scientific and mathematical competencies. It is not an assessment of achievement in the school subjects associated with these domains of knowledge; nevertheless the three main assessment domains of PISA (reading, Mathematics and science) do reflect important desired outcomes of student learning in those curriculum areas. PISA uses the term literacy to emphasise its focus on using knowledge in a variety of situations, and to distinguish it from a focus on measuring achievement against some predefined curriculum syllabus.

> *"I must say the evidence at hand is that 15-year-old students around the world find these kinds of demands very challenging indeed. Students seem unaccustomed to thinking creatively in Mathematics classes. They have a great deal of trouble explaining their thinking and reasoning"*

Regular Learning Curve readers will have seen the article by Juliette Mendelovits in a previous issue (Issue XIII, October 2009) focusing on aspects of the PISA Reading domain. Further information about PISA can be obtained from the most recently published framework document (OECD, 2006). Released PISA survey items are also published in book form (OECD, 2009). These and other publications are available from the OECD website (www.oecd.org).

In the case of PISA Mathematics, survey items are designed to measure proficiencies that reflect the skills needed by mathematically literate individuals in a modern society:

1. the ability to recognise the existence of mathematical elements within a situation presenting a challenge;
2. the ability to pare the situation down to its elements, enabling a separation of those mathematical features from the context, and to define a mathematical problem the resolution of which might help to answer the challenge;
3. the ability to call on relevant mathematical knowledge and to apply that knowledge confidently and correctly to forge a mathematical solution to the problem at hand;
4. the ability to relate that solution back to the original situation, thereby ensuring that the solution makes sense and genuinely addresses the challenge – that is, the ability to recognise the extent and limitations of the solution;
5. the ability to communicate the outcomes to others; and
6. the ability to step outside the process and exercise control mechanisms that help direct thought and action to achieve the desired outcome.

Being mathematically literate in the modern world means much more than memorising rules and formulae and mastering a set of procedures. It also means being able to think creatively about challenges, to analyse situations, to link the demands of those situations with relevant knowledge held by the individual, to apply that knowledge in appropriate ways, and to reflect on and evaluate the solution found and the process followed to reach it.

How much teaching and learning time is devoted to presenting students with problems set in authentic contexts? To what extent does India's education system value the investment of effort by individuals and groups of students in grappling with such problems, pulling a problem apart, hunting for the knowledge that would help to make the problem tractable, imposing an analytical structure, applying Mathematics to the problem, developing solutions, evaluating the solutions, communicating the solutions to others, considering ways of implementing the solutions, and talking about that whole exploration and discovery process?

If that investment of time is not made, is it reasonable to expect students to do well at assessments that call on those skills and processes? Could we expect students to go into post-secondary courses with the confidence and competence to interact productively with problems that demand those skills? Could employers reasonably expect their young workers to have those skills?

I must say the evidence at hand is that 15-year-old students around the world find these kinds of demands very challenging indeed. Students seem unaccustomed to thinking creatively in Mathematics classes. They have a great deal of trouble explaining their thinking and reasoning. They frequently find it very difficult to make decisions about what mathematical knowledge and skills might be relevant to solving a particular problem when that information is not given to them directly. Of course the PISA data show the proportions of the student cohort in each participating country demonstrating various levels of proficiency, and in each country some students are in the highest described level. But many more are in lower levels. Furthermore, recent criticism of the Mathematics component of PISA has been based on the claim that the level of Mathematics required to successfully solve the survey items has been too low.

To illustrate a PISA item that starts in a real world situation, and demands some thought, analysis and interpretation rather than simple application of routine procedural knowledge, I present a test item that was used in a previous PISA survey.

PISA Mathematics uses 'authentic' to refer to problems for which there is genuine interest in the solution and for which the purpose of using Mathematics is to solve the problem, in contrast to contrived applications that are presented mainly for the purpose of practising particular skills. The most authentic problem contexts are those for which the context itself influences the solution and its interpretation, and which therefore require students to consciously and actively connect the problem context with the

Rock Concert, a Mathematics item from the PISA 2003 field trial shown in the box, presents a context that would be familiar to many 15-year-olds, and provides the opportunity to devise a model for the amount of space that a person might occupy while standing. With the multiple choice format used in this item, this could be done by postulating an area for each person, multiplying it by the number of people given in each of the options provided, and comparing the result to the conditions given in the question. Alternatively, the reverse could be done, starting with the area provided and working backwards using each of the response options, to the corresponding space per person, and deciding which one best fits the criteria established in the question. The student must think clearly about the relationship between the model he or she uses and the resulting solution on the one hand, and the real context on the other, in order to validate the model used and to be sure he or she has chosen the most realistic answer.

ROCK CONCERT

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.
Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A 2 000
B 5 000
C 20 000
D 50 000
E 100 000

The level of the mathematical knowledge required to answer this question is not very high, nevertheless this item was not answered correctly by many students. In fact only about 30 per cent of students internationally selected response option C as the most reasonable answer to this question when it was administered in the field trial for PISA 2003. It would seem that the level of the Mathematics required may not be the main issue here – rather, something more fundamental is going wrong. Students are not sufficiently able to think outside the square, to reason and argue, to discuss and explain their thinking.

The PISA focus on mathematical literacy implies a set of teaching and learning objectives that could form an important part of mathematical instruction at least in the junior and middle years of secondary school, and possibly more widely than that. If school Mathematics programs took these objectives to heart, arguably students would be better equipped to make confident and effective use of the mathematical skills they have learned at school as they negotiate the challenges of their life as citizens.

To illustrate a teaching and learning activity that might be useful in a Mathematics classroom to help students think about and analyze real world situations from a mathematical perspective, I revisit an Idea I first presented to the 2005 annual conference of the mathematical Association of Victoria (Turner, 2005). Suppose that watermelons were made to grow in cuboid shapes as shown in the photograph rather than their more familiar ellipsoidal shape. What possibilities and problems would such a change of shape cause for growers, handlers, sellers, and consumers? What mathematical ideas might come from this simple piece of stimulus? Here are some:

1. It is said that this shape was developed to save space. How might this be true? Consider storage, packing and transport of this shape compared to the more common spherical or ellipsoidal watermelons.

2. When watermelons are cut up into serving slices, we usually use sectors of a circle. What shapes might work for a 'cuboid' watermelon? What would be the advantages and disadvantages of different shapes?

3. What is the relationship between surface area and volume for the 'cuboid' watermelon compared to an ellipsoidal melon? Would one of these shapes have more skin, or rind, per kilogram than the other? What would be the relationship between the relative proportion of skin and flesh for the different shapes?

4. What other mathematical ideas does this watermelon context suggest?

There are some additional steps that would be needed if such an idea were to be used as an assessment item. First, experience in using it as part of a teaching and learning activity will tell you more about its potential in assessment. That experience will tell you what questions you could reasonably ask. In addition, it will tell you what range of responses students might give, which you probably need to know if you are to plan a marking scheme in advance. Here are some possibilities:

Q1. Imagine you have a spherical watermelon with diameter 30 cm. What would be the dimensions of a 'cuboid' watermelon with the same volume? (This could be presented as an open question, or perhaps in multiple-choice form using anticipated calculation errors as distracters.)

Q2. If you were able to grow a 'cuboid' watermelon having dimensions of about 30 cm, about how much more flesh would you expect compared to a spherical watermelon with diameter 30 cm?

Show your calculations, and explain and justify your answer.

The increasing popularity and accessibility of digital photography mean that anyone with a little imagination can come up with interesting stimulus ideas from the world around that can form the basis of significant teaching and learning activities and assessment tasks. This might be an easy stepping stone towards providing more opportunities in the classroom for exploring the relationships between Mathematics and real world phenomena, and to developing and practicing the skills mentioned earlier that are so vital for effective participation in an increasingly quantitative world.

[Readers may be interested to learn that India now participates in PISA. Two states (Himachal Pradesh and Tamil Nadu) are taking part in the implementation of the PISA 2009 survey on a one-year-delayed timeline, following implementation by the 66 participants in the main survey during 2009. Results for these Indian states should be published in late 2011.]

References:

1. OECD (2006). Assessing Scientific, Reading and mathematical Literacy: A framework for PISA 2006. Paris: OECD.
2. OECD (2009). Take the test: sample questions from OECD's PISA assessments. Paris: OECD.
3. Turner, Ross (2005). “PISA: Leaning Towards Literacy.” Mathematics Association of Victoria 2005 December Conference Proceedings. Victoria, Australia: Mathematics Association of Victoria.

Ross Turner is a Principal Research Fellow at ACER. He manages ACER's involvement as lead contractor to the OECD responsible for implementing the International PISA project. He is also the Deputy International Project Director for PISA, and the leader of the PISA Mathematics assessment domain. He can be contacted at Turner@acer.edu.au